

1. THE PROBLEM

Let X be a Banach space and $T: X \rightarrow X$ a linear operator. We assume that T is invertible and that T^{-1} is bounded. We define the operator $S: X \rightarrow X$ by $Sx = Tx - x$.

Let $\{x_n\}_{n=0}^{\infty}$ be a sequence in X such that $x_n = S^n x_0$ for all $n \geq 0$. We assume that $\|x_n\| \leq M$ for all $n \geq 0$ and that $\|x_0\| = 1$.

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n	$\ x_n\ $	$\ y_n\ $	$\ z_n\ $
0	1	1	0
1	$\ Tx_0 - x_0\ $	$\ Tx_0\ $	$\ Tx_0 - x_0\ $
2	$\ T^2x_0 - 2Tx_0 + x_0\ $	$\ T^2x_0\ $	$\ T^2x_0 - 2Tx_0 + x_0\ $
3	$\ T^3x_0 - 3T^2x_0 + 3Tx_0 - x_0\ $	$\ T^3x_0\ $	$\ T^3x_0 - 3T^2x_0 + 3Tx_0 - x_0\ $
4	$\ T^4x_0 - 4T^3x_0 + 6T^2x_0 - 4Tx_0 + x_0\ $	$\ T^4x_0\ $	$\ T^4x_0 - 4T^3x_0 + 6T^2x_0 - 4Tx_0 + x_0\ $
5	$\ T^5x_0 - 5T^4x_0 + 10T^3x_0 - 10T^2x_0 + 5Tx_0 - x_0\ $	$\ T^5x_0\ $	$\ T^5x_0 - 5T^4x_0 + 10T^3x_0 - 10T^2x_0 + 5Tx_0 - x_0\ $